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# Utilizing Purcell's Mirror Method with Mismatch

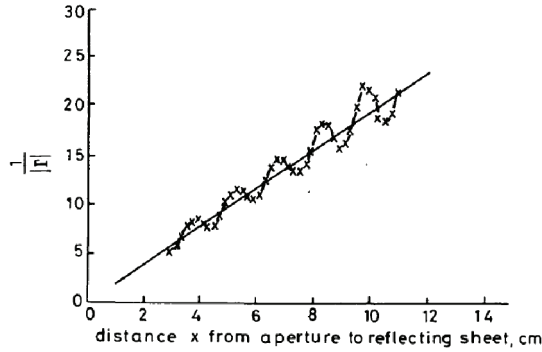
## Application Note

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This note describes making single antenna measurements using the mirror image technique as it applies to the DE Antenna Network and Simulator Math icon. The chamber ripple affect is reduced by applying the filter icon. This method was originally developed by Purcell and later improved to include mismatch by Z. Wu (Wu's work was subsequently simplified by Lee and Baddour of NASA).

When an antenna is imaged to itself using a mirror the S11 variation becomes a function of the distance. This is due to the power received from the mirror by the antenna being reflected back setting up multiple bounce. A typical variation is shown below.

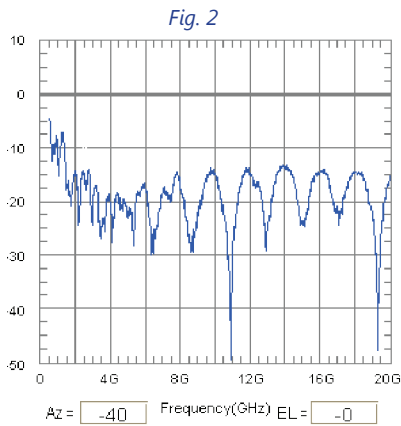


Wu derived the net affect on the S11 pattern above by converging the infinite series of wave bounce to:

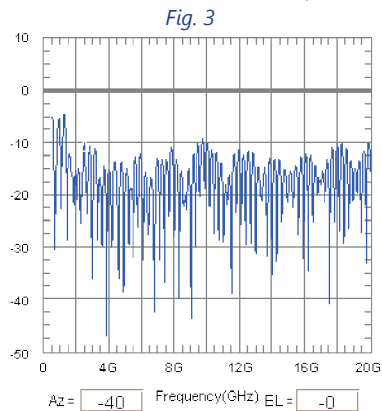
*Fig. 1*

$$\Gamma = \Gamma_0 + \frac{(1 - |\Gamma_0|^2) \left( \frac{\sigma G}{16\pi x^2} \right)^{1/2} \exp(-j2kx)}{1 - \left( \frac{\sigma' G'}{16\pi x^2} \right)^{1/2} \exp(-j2kx)}$$

The DE0726 measurement horn was imaged using the DE0726M mirror. First the horn is pointed into absorber resulting in the following S11 frequency profile:



Then the horn was pointed directly at the mirror at a distance of 0.5 meter resulting in the modulated reflection pattern shown below.



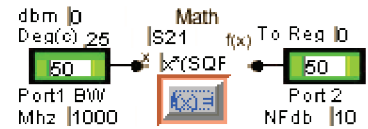
These two patterns establish the basis for determining the gain. Lee and Baddour were able to make some assumptions and simplify the Gain to:

$$G = \frac{|S_{11_M} - S_{11_o}| 8\pi r}{(1 - |S_{11_o}|^2)\lambda}$$

The vector difference of the reflection with and without the mirror is necessary. This means phase must be accurately measured including the path. Equation 2 has been installed into the Math icon and saved in the simulations directory as *Calculates Purcell's Mirror Reflection with Mismatch correction.sch*

Fig. 4

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Standard math operators: +,*,/,sqrt,^,trig,exp example: (x^2)
Enter Text:
x*(SQRT(MAG(reg1-reg2)*8*pi*LWL/(1-MAG(Reg2^2))))
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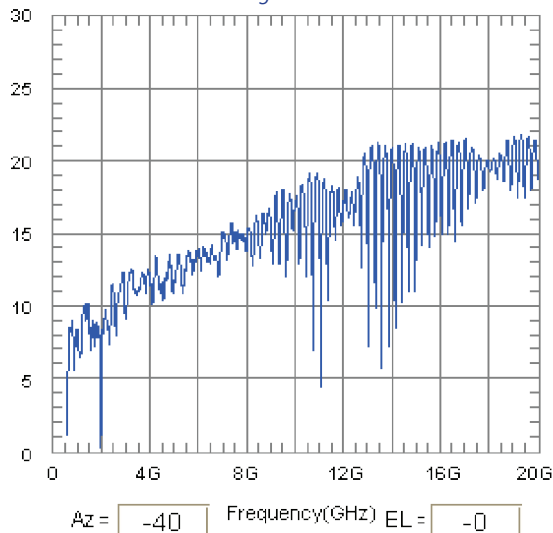
The internal variables Reg1 and Reg2 are used directly. The S11 into the mirror is stored in Reg1 and the S11 into absorber is stored in Reg2. The internal variables are:

**L** = Path length

**WL** = Internal wavelength variable

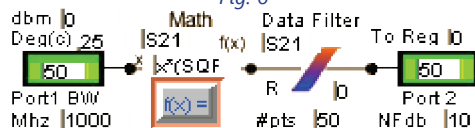
Remembering the S11 is being used as Power transmitted/power received the calculation of S21 is required. Invoking "Calculate S21" results in the following gain profile:

Fig. 5

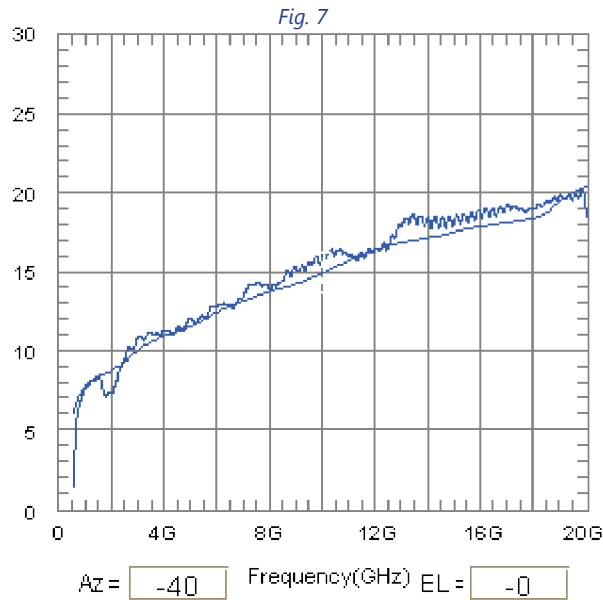


Equation 2 is an approximation to the multi bounce rapidly converging infinite series from which (1) was derived. Data filtering may be applied to the schematic Fig.4. The number of points determines the effective order. 1 point will reproduce the data exactly while larger samples > number for points will over flatten the data.

Fig. 6



The measurement data was done with 1600 frequency points. The data filter is set to filter in groups of 50 points. The resulting gain profile is shown with previous time domain measurements on different DE0726 horn:



Additional data points may be specified however as the path modulation decreases the real data will smooth out and be lost. The gain dip at 2GHz was a known problem which was corrected on the latter unit. # points > 100 is found to impact the real data. While not yet verified the Phase should also be valid.